NAG C Library Function Document

nag_zpotrs (f07fsc)

1 Purpose

nag_zpotrs (f07fsc) solves a complex Hermitian positive-definite system of linear equations with multiple right-hand sides, $AX = B$, where A has been factorized by nag_zpotrf (f07frc).

2 Specification

```
void nag_zpotrs (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer nrhs,
      const Complex a[], Integer pda, Complex b[], Integer pdb, NagError *fail)
```

3 Description

To solve a complex Hermitian positive-definite system of linear equations $AX = B$, this function must be preceded by a call to nag_zpotrf (f07frc) which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If **uplo** = **Nag_Upper**, $A = U^H U$, where U is upper triangular; the solution X is computed by solving $U^H Y = B$ and then $UX = Y$.

If **uplo** = **Nag_Lower**, $A = LL^H$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^H X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether A has been factorized as $U^H U$ or LL^H as follows:

if **uplo** = **Nag_Upper**, then $A = U^H U$, where U is upper triangular;

if **uplo** = **Nag_Lower**, then $A = LL^H$, where L is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: **nrhs** ≥ 0.

5: **a**$[dim]$ – const Complex *Input*

Note: the dimension, dim, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.

On entry: the Cholesky factor of A, as returned by nag_zpotrf (f07frc).

6: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **a**.

Constraint: **pda** $\geq \max(1, \mathbf{n})$.

7: **b**$[dim]$ – Complex *Input/Output*

Note: the dimension, dim, of the array **b** must be at least $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pdb} \times \mathbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, **pdb** $\geq \max(1, \mathbf{n})$;
if **order** = **Nag_RowMajor**, **pdb** $\geq \max(1, \mathbf{nrhs})$.

9: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = $\langle value \rangle$.
Constraint: **n** ≥ 0.

On entry, **nrhs** = $\langle value \rangle$.
Constraint: **nrhs** ≥ 0.

On entry, **pda** = $\langle value \rangle$.
Constraint: **pda** > 0.

On entry, **pdb** = $\langle value \rangle$.
Constraint: **pdb** > 0.

NE_INT_2

On entry, **pda** = $\langle value \rangle$, **n** = $\langle value \rangle$.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = $\langle value \rangle$, **n** = $\langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** $= \langle value \rangle$, **nrhs** $= \langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \textbf{nrhs})$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** = **Nag_Upper**, $|E| \leq c(n)\epsilon|U^H|\,|U|$;

if **uplo** = **Nag_Lower**, $|E| \leq c(n)\epsilon|L|\,|L^H|$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \||A^{-1}|\,|A|\,|x|\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \||A^{-1}|\,|A|\|_\infty \leq \kappa_\infty(A)$. Note that $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_zporfs (f07fvc), and an estimate for $\kappa_\infty(A)$ $(= \kappa_1(A))$ can be obtained by calling nag_zpocon (f07fuc).

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to nag_zporfs (f07fvc) to refine the solution and return an error estimate.

The real analogue of this function is nag_dpotrs (f07fec).

9 Example

To solve the system of equations $AX = B$, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 3.93 - 6.14i & 1.48 + 6.58i \\ 6.17 + 9.42i & 4.65 - 4.75i \\ -7.17 - 21.83i & -4.91 + 2.29i \\ 1.99 - 14.38i & 7.64 - 10.79i \end{pmatrix}.$$

Here A is Hermitian positive-definite and must first be factorized by nag_zpotrf (f07frc).

9.1 Program Text

```
/* nag_zpotrs (f07fsc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer i, j, n, nrhs, pda, pdb;
  Integer exit_status=0;
  Nag_UploType uplo_enum;

  NagError fail;
  Nag_OrderType order;
  /* Arrays */
  char    uplo[2];
  Complex *a=0, *b=0;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
#define B(I,J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07fsc Example Program Results\n\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%*[^\n] ", &n, &nrhs);
#ifdef NAG_COLUMN_MAJOR
  pda = n;
  pdb = n;
#else
  pda = n;
  pdb = nrhs;
#endif

  /* Allocate memory */
  if ( !(a = NAG_ALLOC(n * n, Complex)) ||
       !(b = NAG_ALLOC(n * nrhs, Complex)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A and B from data file */

  Vscanf(" ' %1s '%*[^\n] ", uplo);
  if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
 else if (*(unsigned char *)uplo == 'U')
   uplo_enum = Nag_Upper;
 else
   {
     Vprintf("Unrecognised character for Nag_UploType type\n");
```

```
      exit_status = -1;
      goto END;
   }

  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= n; ++j)
            Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
                  for (j = 1; j <= i; ++j)
                    Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }

  for (i = 1; i <= n; ++i)
    {
            for (j = 1; j <= nrhs; ++j)
              Vscanf(" ( %lf , %lf )", &B(i,j).re, &B(i,j).im);
    }
  Vscanf("%*[^\n] ");

  /* Factorize A */
  f07frc(order, uplo_enum, n, a, pda, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07frc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Compute solution */
  f07fsc(order, uplo_enum, n, nrhs, a, pda, b, pdb, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07fsc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Print solution */
  x04dbc(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs, b, pdb,
         Nag_BracketForm, "%7.4f", "Solution(s)", Nag_IntegerLabels, 0,
         Nag_IntegerLabels, 0, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04dbc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  if (a) NAG_FREE(a);
  if (b) NAG_FREE(b);
  return exit_status;
}
```

9.2 Program Data

```
f07fsc Example Program Data
  4  2                                                  :Values of N and NRHS
  'L'                                                   :Value of UPLO
 (3.23, 0.00)
 (1.51, 1.92) ( 3.58, 0.00)
```

```
(1.90,-0.84) (-0.23,-1.11) ( 4.09, 0.00)
(0.42,-2.50) (-1.18,-1.37) ( 2.33, 0.14) ( 4.29, 0.00)  :End of matrix A
( 3.93, -6.14) ( 1.48,  6.58)
( 6.17,  9.42) ( 4.65, -4.75)
(-7.17,-21.83) (-4.91,  2.29)
( 1.99,-14.38) ( 7.64,-10.79)                           :End of matrix B
```

9.3 Program Results

```
f07fsc Example Program Results

 Solution(s)
                    1                   2
 1  ( 1.0000,-1.0000)  (-1.0000, 2.0000)
 2  (-0.0000, 3.0000)  ( 3.0000,-4.0000)
 3  (-4.0000,-5.0000)  (-2.0000, 3.0000)
 4  ( 2.0000, 1.0000)  ( 4.0000,-5.0000)
```